

MINIMAL SUPERSYMMETRIC HIGGS BOSON DECAY RATE IN $O(\alpha_s^2)$ PERTURBATIVE QCD

Levan R. Surguladze

*Institute of Theoretical Science, University of Oregon
Eugene, OR 97403, USA*

Abstract

A short presentation of the results of the analytical evaluation of the $O(\alpha_s^2)$ QCD contributions in the fermionic decay rates of the neutral CP-odd minimal supersymmetric Higgs particle is made. The corrections due to the nonvanishing quark masses are included. The results are presented both in terms of running and pole quark masses.

A crucial test of the most attractive extension of the Standard Model (SM) - the Minimal Supersymmetric Standard Model (MSSM) can possibly be done at LEP200. Indeed, as was shown in [1], there is a significant mass parameter space increase in the two Higgs doublet model at LEP200, while for the SM Higgs the mass range will increase only marginally. For the MSSM, a large theoretically allowed parameter region can be covered [1] with $\sqrt{s} = 210$ GeV, highest expected luminosity of 500 pb^{-1} and with the latest CDF result for the top mass around 174 GeV [2]. (For earlier analyses of the MSSM Higgs discovery potential at LEP and LHC see [3] and references therein.)

To study the detectability of various Higgs particles, it is important to start with the search of qualitative as well as quantitative information about their decay processes. One of the most interesting experimentally accessible quantities - the branching fractions of the quark-antiquark decay modes can be calculated within the perturbation theory. (For a review of the SM Higgs phenomenology, taking into account radiative effects see [4] and references therein.) A precise theoretical evaluation of decay rates is particularly important from a viewpoint of distinction of MSSM and minimal SM Higgs signals. In the recent work [5] the fermionic decay rates of the minimal SM Higgs particle have been evaluated to the $O(\alpha_s^2)$ in perturbative QCD. In the present work, using the same technique, the $O(\alpha_s^2)$ QCD contributions to the decay

rates of the MSSM pseudoscalar Higgs particle into the quark-antiquark pairs are evaluated analytically, taking into account the nonvanishing quark masses. The assumption was made, that all superpartners are too heavy to appear in the decays of the Higgs boson.

The minimal supersymmetric extension of the SM is the two-Higgs doublet model. The physical Higgs boson spectrum of the MSSM consists of five states: a charged Higgs pair H^\pm , neutral CP -even scalars H_s , H'_s and a neutral CP -odd pseudoscalar H_p (for a review see, e.g., the textbook [6]). The Lagrangian density, describing the Yukawa interaction of the neutral Higgs boson with quarks and leptons has the following form:

$$L = (\sqrt{2}G_F)^{1/2} \sum_f (C_{H_s ff}(\alpha, \beta) m_f \bar{q}_f q_f H_s + C_{H_p ff}(\beta) m_f \bar{q}_f i\gamma_5 q_f H_p) \\ \equiv (\sqrt{2}G_F)^{1/2} \sum_f (j_f^{(s)} H_s + j_f^{(p)} H_p) \quad (1)$$

Below q_f denote the quark fields with flavor f and mass m_f . The coefficients $C_{H_s ff}(\alpha, \beta)$ and $C_{H_p ff}(\beta)$ are the functions of the ratio of the Higgs-field vacuum expectation values $-\tan\beta$ and a mixing angle α in the neutral CP -even sector. The expressions for $C_{H_s ff}(\alpha, \beta)$ and $C_{H_p ff}(\beta)$ for the particular quark or lepton pair can be found, e.g., in [6]. In the SM limit $C_{H_s ff}(\alpha, \beta) = -1$ and the pseudoscalar term drops out in the eq.(1). The corresponding calculation of the $O(\alpha_s^2)$ SM Higgs decay rate (relevant for the MSSM Higgs scalar as well) has been done in [5]. In the present work one considers the pseudoscalar Higgs, expecting that the reader is familiar with the ref.[5].

The two-point correlation function of the pseudoscalar currents $j_f^{(p)}$ has the following form:

$$\Pi(Q^2 = -s, m_f) = i \int e^{iqx} \langle T j_f^{(p)}(x) j_f^{(p)}(0) \rangle_0 d^4x. \quad (2)$$

For the decay width one has:

$$\Gamma_{H_p \rightarrow q_f \bar{q}_f} = \frac{\sqrt{2}G_F}{M_{H_p}} \text{Im} \Pi(s + i0, m_f) \Big|_{s=M_{H_p}^2}. \quad (3)$$

The full $O(\alpha_s)$ analytical result for the decay rate of $H_p \rightarrow q_f \bar{q}_f$ in terms of pole quark masses looks like [7]:

$$\Gamma_{H_p \rightarrow q_f \bar{q}_f} = \frac{3\sqrt{2}G_F M_{H_p}}{8\pi} C_{H_p ff}(\beta) m_f^2 \left(1 - \frac{4m_f^2}{M_{H_p}^2}\right)^{\frac{1}{2}} \left[1 + \frac{\alpha_s(M_{H_p})}{\pi} \delta^{(1)}\left(\frac{m_f^2}{M_{H_p}^2}\right) + O(\alpha_s^2)\right], \quad (4)$$

where:

$$\delta^{(1)} = \frac{4}{3} \left[\frac{a(\eta)}{\eta} + \frac{19 + 2\eta^2 + 3\eta^4}{16\eta} \log \gamma + \frac{21 - 3\eta^2}{8} \right],$$

$$a(\eta) = (1 + \eta^2) \left[4Li_2(\gamma^{-1}) + 2Li_2(-\gamma^{-1}) - \log \gamma \log \frac{8\eta^2}{(1 + \eta)^3} \right] - \eta \log \frac{64\eta^4}{(1 - \eta^2)^3},$$

$$\gamma = \frac{1 + \eta}{1 - \eta}, \quad \eta = \left(1 - \frac{4m_f^2}{M_{H_p}^2} \right)^{\frac{1}{2}}.$$

The expansion of the r.h.s of eq.(4) in a power series in terms of small $m_f^2/M_{H_p}^2$ has the following form:

$$\Gamma_{H_p \rightarrow q_f \bar{q}_f} = \frac{3\sqrt{2}G_F M_{H_p}}{8\pi} C_{H_p ff}(\beta) m_f^2 \left\{ \left(1 - 2\frac{m_f^2}{M_{H_p}^2} + \dots \right) + \frac{\alpha_s(M_{H_p})}{\pi} \left[3 - 2 \log \frac{M_{H_p}^2}{m_f^2} + \frac{m_f^2}{M_{H_p}^2} \left(8 + 8 \log \frac{M_{H_p}^2}{m_f^2} \right) + \dots \right] + O(\alpha_s^2) \right\}, \quad (5)$$

where the period covers high order terms $\sim (m_f/M_{H_p})^{2k}$, $k = 2, 3, \dots$

The evaluation of the $O(\alpha_s^2)$ corrections to the eqs. (4,5) has been done in a close analogy of the similar calculation of the decay rate of SM Higgs boson [5]. Namely, the full two-point correlation function (2) was expanded in powers of m_f^2/Q^2 in the “deep” Euclidean region ($Q^2 \gg m_f^2$). The first two coefficient functions in this expansion was evaluated to $O(\alpha_s^2)$ by applying an appropriate projectors to the $\Pi(Q^2, m_f^B, m_v^B)$. m_v is the mass of the quark appearing virtually in some topological types of three-loop diagrams and “B” labels the unrenormalized quantities. In the calculation of divergent Feynman integrals the dimensional regularization formalism [8] and the minimal subtraction prescription [9] in its modified form - \overline{MS} [10] are used. The γ_5 matrix is defined in D -dimensional space-time as an object with the following properties:

$$\{\gamma_5, \gamma_\mu\} = 0, \quad \gamma_5 \gamma_5 = 1.$$

The above definition causes no problems in dimensional regularization when there are two γ_5 matrices in a closed fermionic loop. The corresponding one-, two- and three-loop diagrams have been evaluated analytically, using the special computer program HEPLoops [11]. The graph-by-graph results were summed up with an appropriate symmetry and gauge group weights. In the obtained expressions for the Π_i in terms of bare quantities one renormalizes the coupling and the quark mass in a standard way (see, e.g., [5]). Finally, one should analytically continue the obtained Π_i from Euclidean to Minkowski space and take the imaginary part at $s = M_{H_p}^2$ (eq.(3)). One obtains the following analytical result for the standard QCD with $SU_c(3)$ gauge group:

$$\begin{aligned}
\Gamma_{H_p \rightarrow q_f \bar{q}_f} = & \frac{3\sqrt{2}G_F M_{H_p}}{8\pi} C_{H_p f f}(\beta) m_f^2 \left\{ 1 + \frac{\alpha_s}{\pi} \left(\frac{17}{3} + 2 \log \frac{\mu_{\overline{MS}}^2}{M_{H_p}^2} \right) \right. \\
& + \left(\frac{\alpha_s}{\pi} \right)^2 \left[\frac{10801}{144} - \frac{19}{2} \zeta(2) - \frac{39}{2} \zeta(3) + \frac{106}{3} \log \frac{\mu_{\overline{MS}}^2}{M_{H_p}^2} + \frac{19}{4} \log^2 \frac{\mu_{\overline{MS}}^2}{M_{H_p}^2} \right. \\
& \quad \left. - N \left(\frac{65}{24} - \frac{1}{3} \zeta(2) - \frac{2}{3} \zeta(3) + \frac{11}{9} \log \frac{\mu_{\overline{MS}}^2}{M_{H_p}^2} + \frac{1}{6} \log^2 \frac{\mu_{\overline{MS}}^2}{M_{H_p}^2} \right) \right] \\
& \quad \left. - \frac{m_f^2}{M_{H_p}^2} \left\langle 2 + \frac{\alpha_s}{\pi} \left(\frac{8}{3} + 8 \log \frac{\mu_{\overline{MS}}^2}{M_{H_p}^2} \right) \right. \right. \\
& \quad + \left(\frac{\alpha_s}{\pi} \right)^2 \left[\frac{1429}{36} - 54 \zeta(2) - \frac{166}{3} \zeta(3) + \frac{155}{3} \log \frac{\mu_{\overline{MS}}^2}{M_{H_p}^2} + 27 \log^2 \frac{\mu_{\overline{MS}}^2}{M_{H_p}^2} \right. \\
& \quad \left. \left. - N \left(\frac{3}{2} - \frac{4}{3} \zeta(2) - \frac{4}{3} \zeta(3) + \frac{14}{9} \log \frac{\mu_{\overline{MS}}^2}{M_{H_p}^2} + \frac{2}{3} \log^2 \frac{\mu_{\overline{MS}}^2}{M_{H_p}^2} \right) \right] \right\rangle \\
& \left. + \left(\frac{\alpha_s}{\pi} \right)^2 \sum_{v=u,d,s,c,b} \frac{m_v^2}{M_{H_p}^2} 4 \right\}, \tag{6}
\end{aligned}$$

where the Riemann functions $\zeta(2) = \pi^2/6$ and $\zeta(3) = 1.202056903$. The last term in eq.(6) represents the contributions from the three-loop diagrams containing the virtual quark loop (see fig.1 of the ref.[5]). The “triangle anomaly” type correction (see fig.2 of the ref.[5]) requires a special treatment because of γ_5 in dimensional regularization, and is not included here. However, it is reasonable to expect that their contribution will not exceed 1%. Note that the “triangle anomaly” type contributions vanish identically in the zero quark mass limit. Note also the smallness of corresponding corrections in the scalar channel [5].

The leading three-loop term $\sim m_f^2$ (the massless approximation) coincides with the one obtained in [12, 13, 5], while the three-loop results $\sim m_f^4, m_f^2 m_v^2$ are new.

The calculated decay width obeys the homogeneous renormalization group equation:

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \alpha_s \frac{\partial}{\partial \alpha_s} - \gamma_m(\alpha_s) \sum_{l=f,v} m_l \frac{\partial}{\partial m_l} \right) \Gamma_{H_p \rightarrow q_f \bar{q}_f} \left(\frac{\mu^2}{M_{H_p}^2}, m_f, m_v, \alpha_s \right) = 0. \tag{7}$$

For the standard definition of the QCD β -function and the mass anomalous dimension see the, e.g., ref.[5] (eqs. (14)-(17)). The renormalization group relates the coefficients of log terms in eq.(6). The corresponding relations up to $O(\alpha_s^3)$ for the SM Higgs decay, which are valid for the pseudoscalar Higgs as well, has been obtained in [5].

The solution of the renormalization group equation (7) at $\mu_{\overline{MS}}^2 = M_{H_p}^2$ has the following form:

$$\begin{aligned}
\Gamma_{H_p \rightarrow q_f \bar{q}_f} = & \frac{3\sqrt{2}G_F M_{H_p} C_{H_p f f}(\beta) m_f^2(M_{H_p})}{8\pi} \left\{ 1 - 2 \frac{m_f^2(M_{H_p})}{M_{H_p}^2} + \frac{\alpha_s(M_{H_p})}{\pi} \left(5.66667 - \frac{8}{3} \frac{m_f^2(M_{H_p})}{M_{H_p}^2} \right) \right. \\
& + \left(\frac{\alpha_s(M_{H_p})}{\pi} \right)^2 \left[35.93996 - 1.35865N + \frac{m_f^2(M_{H_p})}{M_{H_p}^2} \left(115.64581 - 2.29599N \right) \right. \\
& \left. \left. + 4 \sum_{v=u,d,s,c,b} \frac{m_v^2(M_{H_p})}{M_{H_p}^2} \right] \right\}. \tag{8}
\end{aligned}$$

For the standard parametrization of the running coupling and the running mass see, e.g., ref.[5]. One can see that the three-loop mass correction is positive in contrary to the analogous result for the SM Higgs [5].

The relation between the \overline{MS} quark mass $m_f(M)$ renormalized at arbitrary M and evaluated for the N -flavor theory and the pole quark mass m_f has the following form [5]:

$$\begin{aligned}
m_f^{(N)}(M) = m_f \left\{ 1 - \frac{\alpha_s^{(N)}(M)}{\pi} \left(\frac{4}{3} + \gamma_0 \log \frac{M^2}{m_f^2} \right) - \left(\frac{\alpha_s^{(N)}(M)}{\pi} \right)^2 \left[K_f \right. \right. \\
+ \sum_{m_f < m_{f'} < M} \delta(m_f, m_{f'}) - \frac{16}{9} + \left(\gamma_1^{(N)} - \frac{4}{3} \gamma_0 + \frac{4}{3} \beta_0^{(N)} \right) \log \frac{M^2}{m_f^2} \\
\left. \left. + \frac{\gamma_0}{2} (\beta_0^{(N)} - \gamma_0) \log^2 \frac{M^2}{m_f^2} \right] \right\}, \tag{9}
\end{aligned}$$

where $\beta_0^{(N)}$, γ_0 and $\gamma_1^{(N)}$ are the perturbative coefficients of the QCD β function and the mass anomalous dimension γ_m correspondingly and can be found, e.g., in ref.[5]. K_f and $\delta(m_f, m_{f'})$ can be obtained from the on-shell results of [14] (see [5]):

$$K_f = \frac{3817}{288} + \frac{2}{3}(2 + \log 2)\zeta(2) - \frac{1}{6}\zeta(3) - \frac{N_f}{3} \left(\zeta(2) + \frac{71}{48} \right) + \frac{4}{3} \sum_{m_l \leq m_f} \Delta\left(\frac{m_l}{m_f}\right), \tag{10}$$

$$\delta(m_f, m_{f'}) = -\frac{1}{3}\zeta(2) - \frac{71}{144} + \frac{4}{3}\Delta\left(\frac{m_{f'}}{m_f}\right), \tag{11}$$

$$\Delta(r) = \frac{1}{4} \left[\log^2 r + \zeta(2) - \left(\log r + \frac{3}{2} \right) r^2 - (1+r)(1+r^3)L_+(r) - (1-r)(1-r^3)L_-(r) \right], \tag{12}$$

$$L_{\pm}(r) = \int_0^{1/r} dx \frac{\log x}{x \pm 1}.$$

$L_{\pm}(r)$ can be evaluated for different quark mass ratios r numerically (see the table 1 of the ref.[5]). In the above equations the number of participating quark flavors N is specified according to the size of M and has no correlation with the quark mass m_f .

Substituting eqs. (9)-(12) at $M = M_{H_p}$ and the coefficients of the renormalization group functions into the eq.(8), one obtains the general form for the decay rate $\Gamma_{H_p \rightarrow q_f \bar{q}_f}$ in terms of the pole quark masses:

$$\begin{aligned}
\Gamma_{H_p \rightarrow q_f \bar{q}_f} = & \frac{3\sqrt{2}G_F M_{H_p}}{8\pi} C_{H_p ff}(\beta) m_f^2 \left\{ 1 - 2 \frac{m_f^2}{M_{H_p}^2} \right. \\
& + \frac{\alpha_s^{(N)}(M_{H_p})}{\pi} \left[3 - 2 \log \frac{M_{H_p}^2}{m_f^2} + \frac{m_f^2}{M_{H_p}^2} \left(8 + 8 \log \frac{M_{H_p}^2}{m_f^2} \right) \right] \\
& + \left(\frac{\alpha_s^{(N)}(M_{H_p})}{\pi} \right)^2 \left\langle \frac{697}{18} - \left(\frac{73}{6} + \frac{4}{3} \log 2 \right) \zeta(2) - \frac{115}{6} \zeta(3) \right. \\
& \quad \left. - N \left(\frac{31}{18} - \zeta(2) - \frac{2}{3} \zeta(3) \right) \right. \\
& + \frac{m_f^2}{M_{H_p}^2} \left[45 + \left(\frac{194}{3} + \frac{16}{3} \log 2 \right) \zeta(2) + 54 \zeta(3) - N \left(\frac{22}{9} + 4 \zeta(2) + \frac{4}{3} \zeta(3) \right) \right] \\
& \quad - \left[\frac{87}{4} - \frac{13}{18} N - \frac{m_f^2}{M_{H_p}^2} \left(31 - \frac{26}{9} N \right) \right] \log \frac{M_{H_p}^2}{m_f^2} \\
& \quad - \left[\frac{3}{4} - \frac{1}{6} N + \frac{m_f^2}{M_{H_p}^2} \left(5 + \frac{2}{3} N \right) \right] \log^2 \frac{M_{H_p}^2}{m_f^2} \\
& \quad \left. - \left(\frac{8}{3} - \frac{32}{3} \frac{m_f^2}{M_{H_p}^2} \right) \sum_{m_l < M_{H_p}} \Delta \left(\frac{m_l}{m_f} \right) + 4 \sum_{m_v < M_{H_p}} \frac{m_v^2}{M_{H_p}^2} \right\rangle \left. \right\}. \tag{13}
\end{aligned}$$

The above result confirms the asymptotic form (5) of the two-loop exact result (4), while the $O(\alpha_s^2)$ expression is new.

The numerical values of $\Delta(m_l/m_f)$, defined in the eq.(12), are given in the table 1 of the ref.[5]. The quark masses are estimated from QCD sum rules: $m_b = 4.72$ GeV [15], $m_c = 1.46$ GeV [16], $m_s = 0.27$ GeV [17], $m_u + m_d \approx m_s/13 = 0.02$ GeV [18].

For the decay mode $H_p \rightarrow b\bar{b}$ with five participating quark flavors one obtains numerically:

$$\begin{aligned}
\Gamma_{H_p \rightarrow b\bar{b}} = & \frac{3\sqrt{2}G_F M_{H_p}}{8\pi} C_{H_p ff}(\beta) m_b^2 \left\{ \left(1 - \frac{4m_b^2}{M_{H_p}^2} \right)^{\frac{1}{2}} + \frac{\alpha_s^{(5)}(M_{H_p})}{\pi} \delta^{(1)} \left(\frac{m_b^2}{M_{H_p}^2} \right) \left(1 - \frac{4m_b^2}{M_{H_p}^2} \right)^{\frac{1}{2}} \right. \\
& + \left(\frac{\alpha_s^{(5)}(M_{H_p})}{\pi} \right)^2 \left[-2.23039 + 169.22983 \frac{m_b^2}{M_{H_p}^2} \right. \\
& - \left(18.13889 - 16.55556 \frac{m_b^2}{M_{H_p}^2} \right) \log \frac{M_{H_p}^2}{m_b^2} + \left(0.08333 - 8.33333 \frac{m_b^2}{M_{H_p}^2} \right) \log^2 \frac{M_{H_p}^2}{m_b^2} \\
& \quad \left. \left. - \left(2.66667 - 10.66667 \frac{m_b^2}{M_{H_p}^2} \right) \sum_{m_l \leq m_b} \Delta \left(\frac{m_l}{m_b} \right) + 4 \sum_{m_v \leq m_b} \frac{m_v^2}{M_{H_p}^2} \right] \right\}, \tag{14}
\end{aligned}$$

where $\delta^{(1)}$ is defined in eq.(4).

In ref.[5] it was observed that the high order corrections reduce the scale dependence significantly and resolve the large discrepancy between the results for the decay rate of SM Higgs in terms of running and pole quark masses. The theoretical uncertainty was estimated at 5%. The same effects were observed for the decay rate of pseudoscalar Higgs particle. In the present calculation one also estimates the theoretical uncertainty at approximately 5%.

It should be stressed, that the virtual top quark, which appears in some topological types of three-loop diagrams (see fig.1 and fig.2 in ref.[5]) may give a nonnegligible contribution. In addition, one may include a heavy virtual superpartners. However, the superpartners decouple [19] if the Higgs mass is much smaller than the supersymmetry scale.

Acknowledgments It is a pleasure to thank D.Soper for discussions, D.Broadhurst and B.Kniehl for helpful communications. This work was supported by the U.S. Department of Energy under grant No. DE-FG06-85ER-40224.

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